Wavelets Application to Study the Bedforms of Parana River

Ronald R. Gutierrez[1], Jorge D. Abad[2], Daniel Parsons[3], Jim Best[4] and Oscar Orfeo[5]

[1]Earth Processes & Environmental Flows, School of Civil and Environmental Engineering, University of Pittsburgh, 963 Benedum Hall; email: rrg11@pitt.edu
[2]Earth Processes & Environmental Flows, School of Civil and Environmental Engineering, University of Pittsburgh, 943 Benedum Hall; email: jabad@pitt.edu
[3]School of Earth and Environment of the University of Leeds, LS2 9JT, UK; email: d.parsons@leeds.ac.uk
[4]Center for Water as Complex Environmental System, Geology Department of the University of Illinois at Urbana-Champaign; email: jimbest@uiuc.edu
[4]Centro de Ecología Aplicada del Litoral (CECOAL-CONICET), Corrientes, Argentina; email: orfeo@amet.com.ar

ABSTRACT

The configuration and behavior of river bedforms is strongly governed by the interrelationship between present sediment load and hydraulic conditions. A quantitative description of both configuration and behavior is necessary to understand and establish the effect they bring on bedform migration and roughness changes. This contribution elaborates on the application of one-dimensional wavelet transforms to describe the bed morphology of natural channels. Since the election of the mother wavelet is crucial to retrieve representative information of the signal, this contribution also provides insights in the criteria to choose the most efficient mother wavelet based on sampling frequency and signal-to-noise ratios quantities. Likewise, 1D-wavelet is applied to quantify the bedform features of the Parana River. Since bedforms in natural channels predominantly show three-dimensional patterns, we discuss the limitations of one-dimensional wavelet transforms to capture such patterns.

1. Introduction

Fourier transform has been widely used in the analysis of geophysical parameters in recent years to identify peaks and singularities of stationary data but has proved to be limited to locate those peaks and singularities spatially or temporarily.

Wavelet transforms (WT) were developed to overcome such limitations of the Fourier transforms. Although they are relatively new tools in Geophysical Sciences, in hydrology and hydraulics, for instance, WT have been applied to fluid mechanics with isolation of coherent structures in turbulent flows, meteorology with temporal variability of coherent convective storm structures, climatology with long-term land temperature series, paleoclimatology with oxygen isotopic ratios from marine sediments, among others (Labat, 2005). Some applications of 1D-wavelets in sedimentology encompass streamflow and sediment loads temporal variations (Rossi et al, 2009), characterization of bedform morphology (Catano-Lopera et al, 2009), planimetric patterns of meandering rivers (Abad, 2010), seabed morphology pattern recognition (Little, 1994), riverbed roughness (Nyander et al, 2003). By conducting laboratory experiments Catano-Lopera et al. (2009) identified two-dimensional and three-dimensional ripple patterns that were analyzed by applying WT. WT efficiently described the dominant wavelength, and located and quantified agradded and eroded areas. As part of the research objectives of the Earth Processes and Environmental Flows research group of the University of Pittsburgh, the applications of one-dimensional and two-dimensional WT to analyze bedforms are being studied.

The present contribution elaborates the criteria to select the more efficient WT on the basis of sampling frequency (SF) and the signal-to-noise ratio (SNR). Since bedforms in natural channels are predominantly three-dimensional (Parsons et. al., 2005), the limitations of one-dimensional WT to recover such pattern and the methodology to analyze the bedforms of the Parana River is also discussed.

2. wavelet transforms

The one-dimensional WT of a signal $f(x) \in L_2(\Re)$ is obtained by the convolution of the signal and the wavelet function (WF) or mother wavelet $\psi(x)$. The following equation shows the mathematical definition of the WT. There, s and b are the scaling and dilation parameters, respectively; and the bar sign represents the complex conjugate.

$$T(s,b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(x)\bar{\psi}\left(\frac{x-b}{s}\right) dx, s > 0$$

WT can be generally classified as discrete and continuous. The former is the analog of the Discrete Fourier Transform (applicable to inputs that are often created by sampling a continuous function) and is more appropriate for data compression and signal reconstruction, the latter is analogous to the Fourier transform and is usually applied to the analysis and detections of signal singularities and patterns (Antonie et al, 2004).

The more widely used continuous WFs are Morlet and the n-th derivatives of the Gaussian (DOG). Among the DOGs, the Ricker or so-called Mexican hat wavelet represents the second derivative. Among the discrete WF, the Haar and the Daubechies family of n-th order stand out.

One of the key steps to get good results in using the WT is the election of the appropriate WF. Different categories of wavelet and various types of wavelets within each category provide a multitude of options to choose from when analyzing a process of interest (Kumar et al, 1997). The election of the appropriate WF depends on the mathematical and the physical nature of the parameter to be analyzed. Likewise, the selected WF should provide the minimum area of the Heisenberg cell. According to the uncertainty Heisenberg's principle, there is a lower limit to the product of frequency and time resolution. Thus, as time resolution is improved, frequency resolution degrades and vice versa (Addison, 2004).

In order to show the weaknesses and strengths of the continuous WF, a set of synthetic signals were produced. Firstly, a combination of sines and cosines were considered; further, a random term generated by using the Wichman-Hill algorithm was introduced (see Figure 1). All signals were analyzed by a modified version of the wavelet software of Torrence and Compo (1998). The SF values ranged from Nyquist ratios (NR) less than 0.5 up to thousands, and RSRs less than 3 which are considered to be moderate values (Ge, 2007).

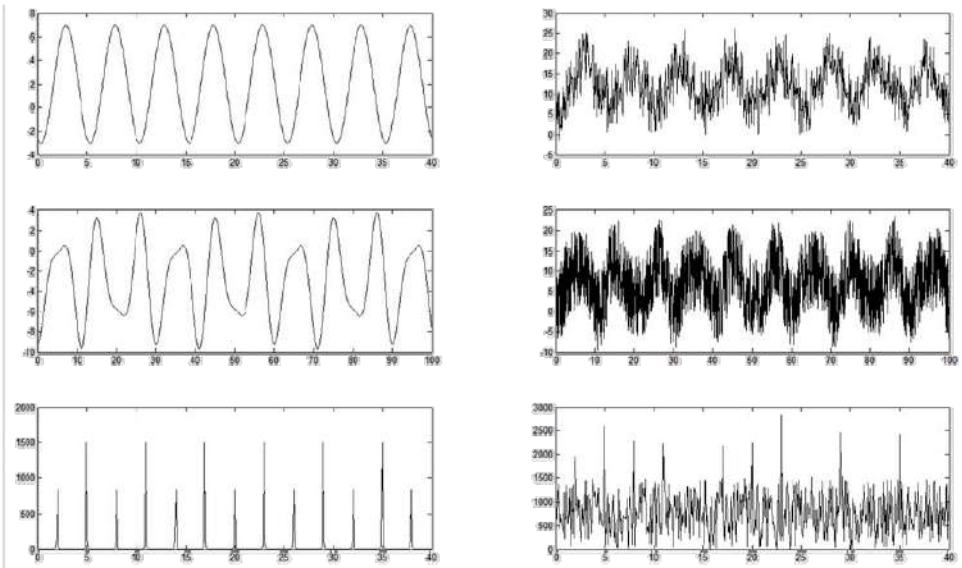

Figure 1. Some examples of synthetic signals (left) and their corresponding noisy versions (right) that were used in the analysis.

Table 1 shows the mathematical expressions that define Ricker and Morlet WF and their respective Fourier transforms. Note that Morlet WF is a complex function; this fact provides it and advantage that is discuss further. Generally, for Morlet WF a value of k_0 of 5 or bigger is assumed to satisfy the admissibility condition (Adisson, 2005).

Table 1. Wavelet functions and their Fourier transforms

Type	Wavelet function	Fourier transform of the WF
Ricker (Mexican Hat)	$\psi(x) = (1 - x^2)e^{-x^2/2}$	$\hat{\psi}(\xi) = (2\pi\xi)^2 e^{-2\pi^2\xi^2}$
Morlet	$\psi(x) = e^{ik_0x}e^{-x^2/2}$	$\hat{\psi}(\xi) = \sqrt{2\pi}e^{(2\pi\xi - k_0)^2/2}$

According to the results of the analysis, we can state the following:

- As expected, no information can be retrieved from the signals for NR less than 0.5. For relatively lower NRs (values of NR less than 10), Morlet WF recovers part of the representative frequencies of the signal and that is not the case when DOGs are used. For relatively higher NRs (values bigger than 50) Morlet WF retrieves coherent and non-coherent frequencies. The efficiency of the DOGs improves as the order of the derivative is increased;

- Morlet WF recovers more information than the DOGs no matter how high is the level of the RSR. For noisy signals, the efficiency of the DOGs improves as the order of the derivative is increased.

These results coincide with some observations of Mi et al (2004), in which it is stated that Morlet WF provides better detection and localization of some ecological patterns. The efficiency of the Morlet WF lays in the fact that it is a complex function whose real part is a rapidly decreasing damped cosine function.

3. Application of Wavelets to Bedforms Studies

Riverbed morphology studies reveled that ripples show two-dimensional (2D) and three-dimensional (3D) patterns depending on their relative location on dunes and the hydraulic patterns. Usually, 3D ripple patterns are observed among troughs and crests although predominantly near to the crests. Likewise, dunes show 3D patterns that are intimately linked to the morphology of the upstream dune, with changes in crest line curvature and crest line bifurcations/junctions significantly influencing the downstream dune form (Parsons et al, 2005). Therefore, an appropriate description of the ripples (either they present 2D patterns or 3D patterns) is key because they affect the shear stress induced by the bedforms to the flow field (Catano-Lopera et. al., 2009).

By conducting laboratory experiments Catano-Lopera et al. (2009) identified the 2D and 3D ripple patterns shown in Figure 2. They were analyzed by applying Morlet WT which efficiently described the dominant wavelength as well as locating and quantifying agradded and eroded areas. Figure 3 shows the spectrogram of the bedforms.

4. Analysis of Bedforms of Parana River

Parsons et. al. (2005) carried out a field study of a swath of 3D dunes in the Rio Parana, Argentina. They surveyed the river bed by using a multibeam echo sounder; and simultaneously, they obtained 3D flow information with an acoustic Doppler current profiler. This study suggests that dune three-dimensionality is intimately connected to the morphology of the upstream dune, with changes in crest line curvature and crest line bifurcations and/or junctions significantly influencing the downstream dune form. The authors of this study also stress the fact that studies over 2D forms neglect the significant influence that lateral flows and secondary circulation may have on the flow structure and thus dune morphology.

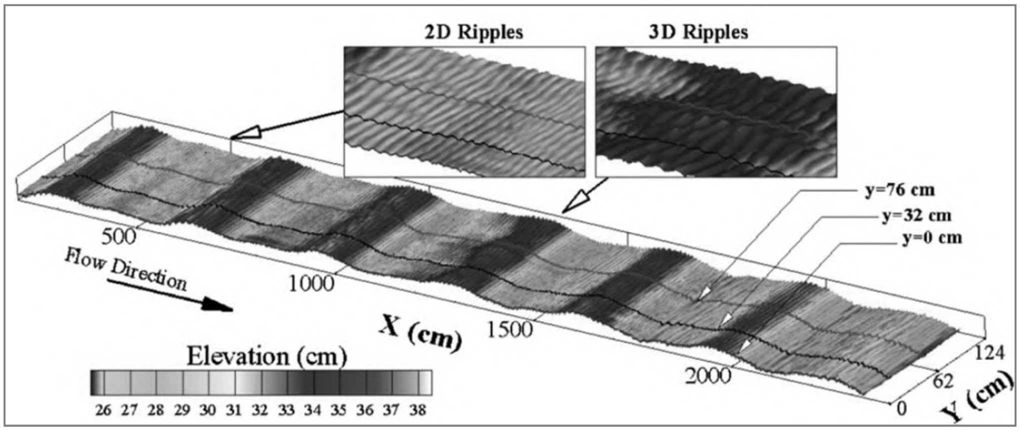

Figure 2. Bedform data used in the study carried out by Catano et al (2009).

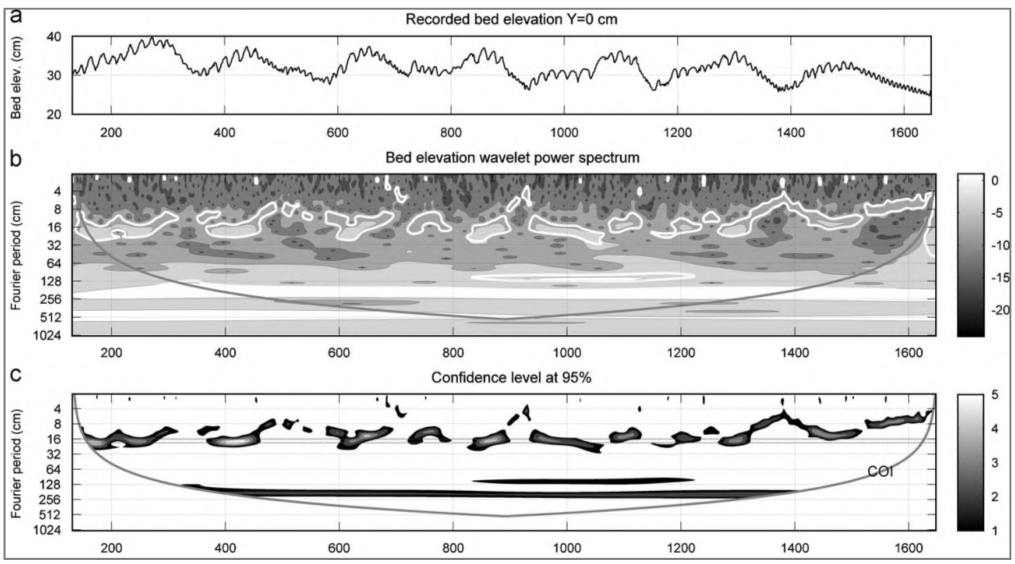

Figure 3. Output of the Morlet WT analysis (3a) Section along the channel stretch, (3b) Morlet wavelet spectrogram, and (3c) 95 percent confidence Morlet wavelet spectrogram.

Parsons et al (2005) obtained continuous data of the morphology of the Parana River of a 200-m width and 1-km length area, approximately (see Figure 4). As shown in Figure 4, three streamwise transects were defined to carry out a wavelet analysis. Likewise, it shows that the morphology of the Parana River presents markedly 3D patterns with bifurcations and crests that do not follow a unique orientation.

As mentioned above, Morlet WF is the most efficient WF to retrieve the dominant frequencies of a signal; therefore, to carry out the analysis of the transects, the use of the Morlet WF was prioritized. Figures 5, 6 and 7 show the results of such analysis.

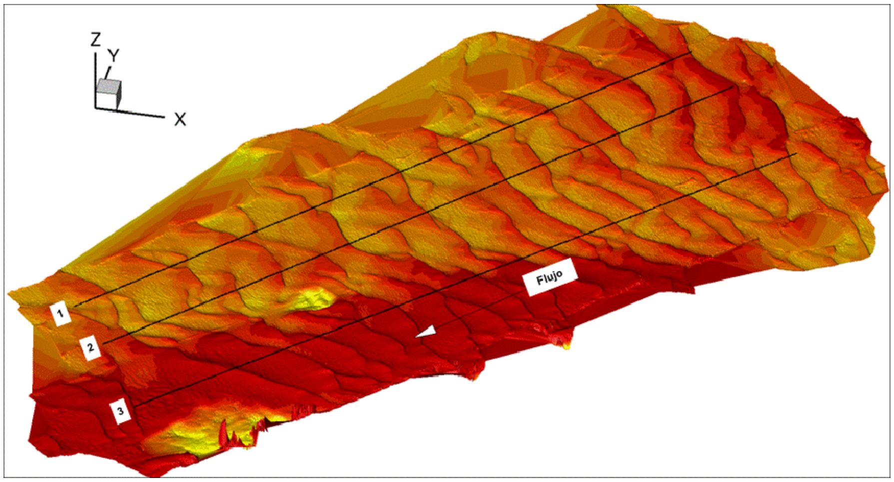

Figure 4. **Riverbed features of River Parana (bathymetry data encompass an area of 200 m x 1 km, approximately. darker areas indicate shallower areas). Note the 3D configuration of the dunes and the skewed pattern of some of them.**

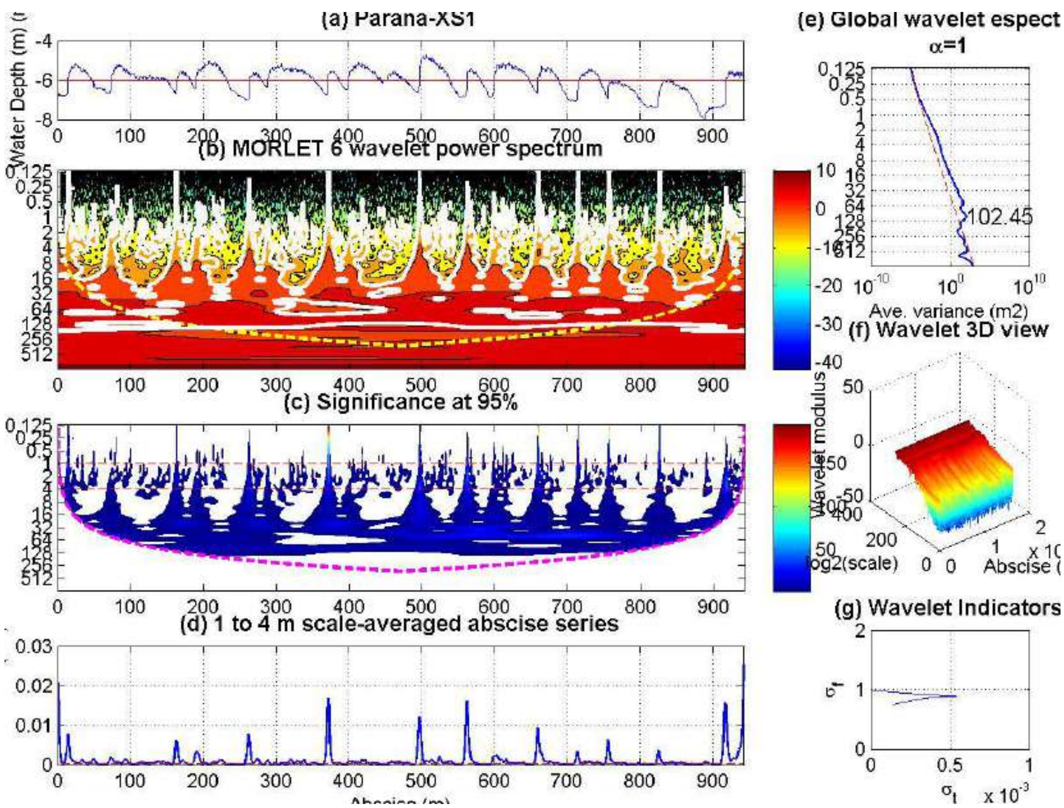

Figure 5. **Output of the wavelet analysis for Transect 1.**

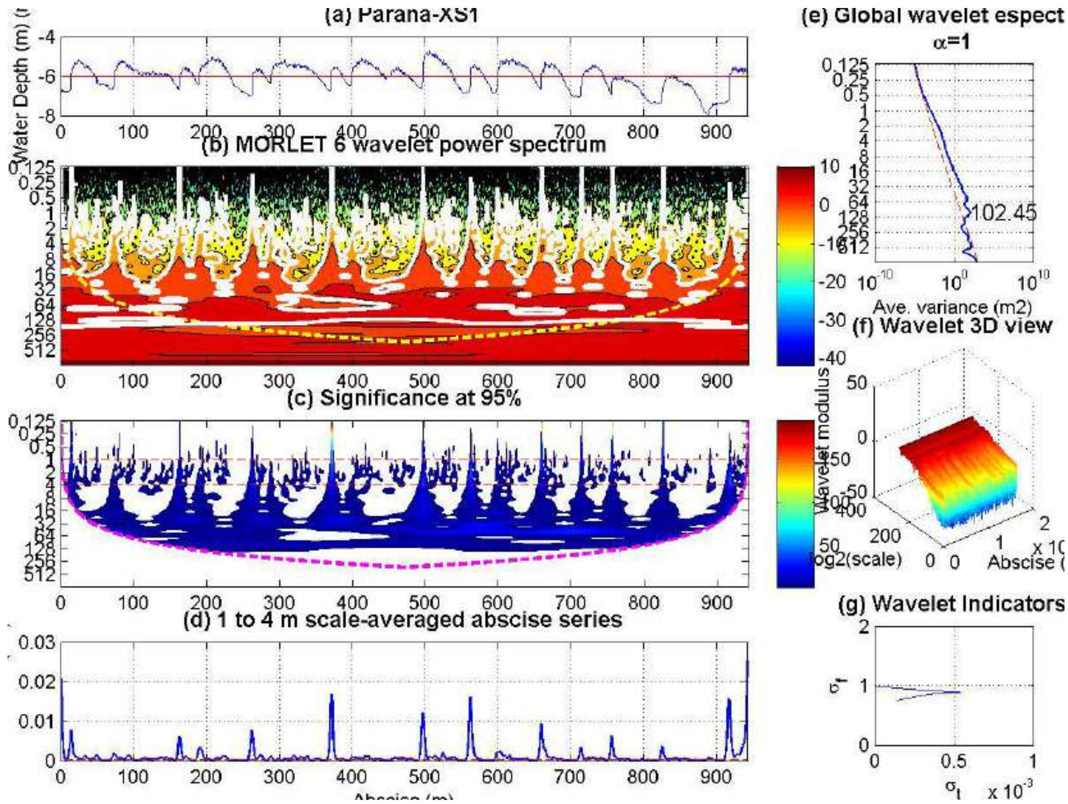

Figure 6. **Output of the wavelet analysis for Transect 2.**

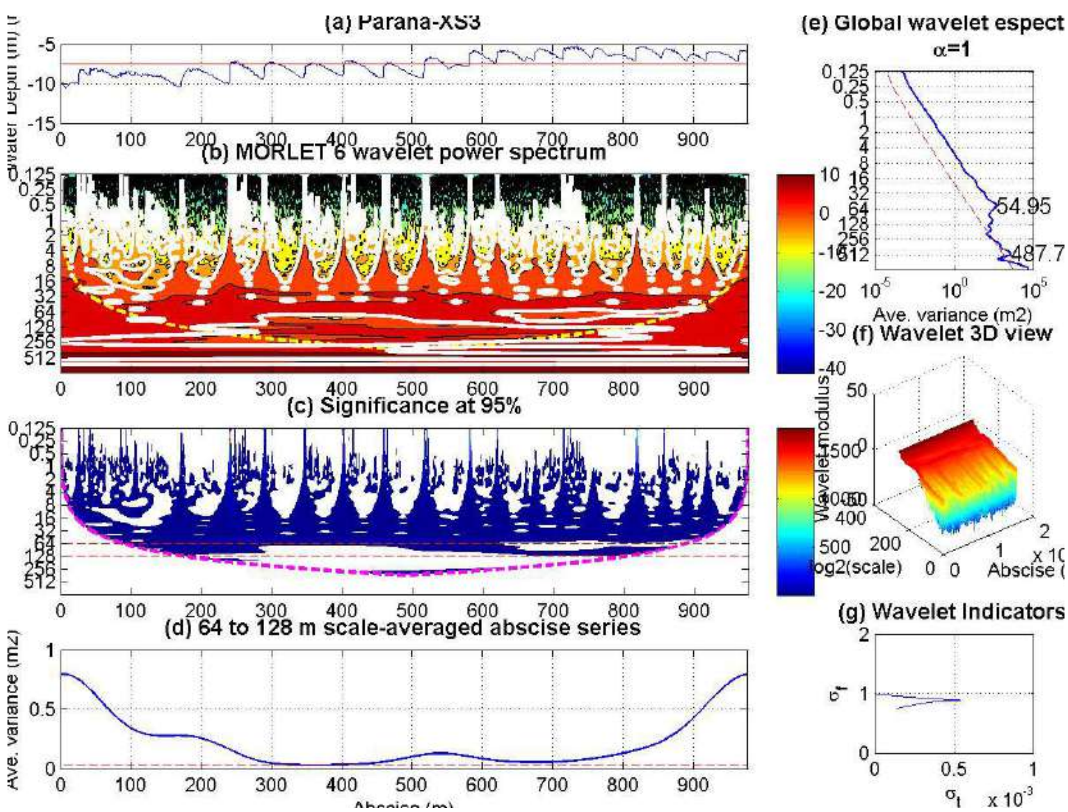

Figure 7. **Output of the wavelet analysis for Transect 3.**

From the results the draw the following observations:

- WTs efficiently quantify predominant wavelengths of the dunes; however, they do not recover that of the ripples. So, a complementary filter should be used to retrieve such information;

- In transects 1 and 2 the wavelet analysis identified just a coherent frequency (which implies that the signals of those transects are relatively stationary) and quasi constant non-coherent frequencies that represent the wavelength of the ripples. That is not the case for transect 3 which shows two predominant frequencies. These facts drive us to deduce that the results of the analysis are biased and that one-dimensional wavelets show limitations to recover information of the bifurcations and the changes of orientation of the dunes and ripples;

- Comparing the results we obtain against that of similar studies carried out by Jain and Keneddy (1971) and Perron et al (2008), in which a multivariate analysis were performed, it is noted that WT overcome the limitations of information provided by the spectrograms. Multivariate analysis just provides a characteristic wavelength and assumes a priori that the signal is stationary.

5. conclusions

One-dimensional wavelets do not efficiently describe the spatial configuration of bedform wavelengths when they are markedly three-dimensional. We foresee that two-dimensional wavelets could overcome such limitation of the one-dimensional wavelets. Two-dimensional wavelets, in time, could quantify more precisely the roughness, sheer stresses and grain sheer stresses in the study stretch of the Parana River.

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